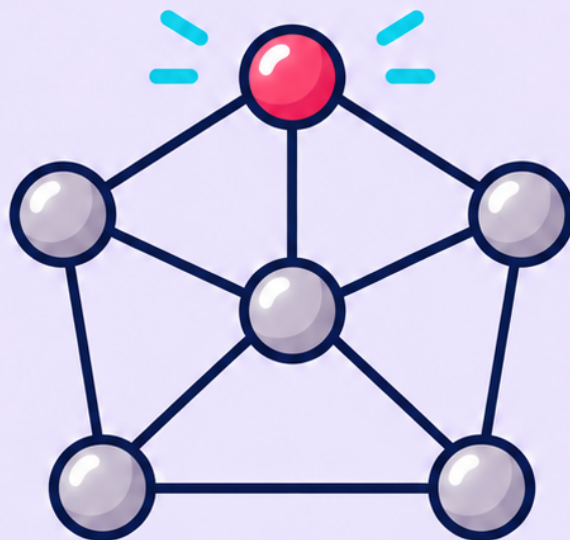


Sum+hingelse

BEYOND CURRICULUM

Graphs

What is a graph?



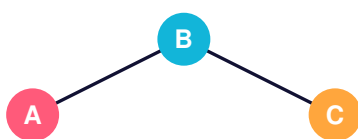
Lesson 1

What is a graph?

Start with the simplest idea: dots joined by lines.

A graph is just dots and lines

A graph is made of dots and lines. In graph theory, the dots are called **vertices** and the lines joining them are called **edges**. The exact position of the dots usually does not matter. What matters is which vertices are connected to which.



TRY IT

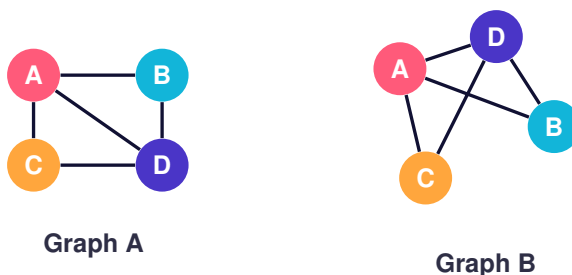
Quick check

How many vertices does this graph have? How many edges?

[Check the answers](#)

Same graph, different picture

Moving the vertices around, changing the lengths of the edges, or even making edges cross does not usually change the graph. The pattern of connections is what matters.



TRY IT

Same graph?

Do Graph A and Graph B represent the same graph? Explain how you can tell.

CONTINUED

[Check the answers](#)

DEFINITION

Degree of a vertex

The **degree** of a vertex is the number of edges that meet at that vertex.

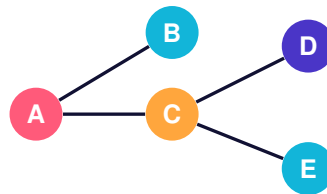
Tip Learning tip

After reading a definition, make up one example and one non-example. An “almost right” example is especially useful: what single feature makes it fail?

The handshake puzzle

Imagine a party. Draw one vertex for each person and one edge whenever two people shake hands. Suppose the handshakes are:

Alice–Ben, Alice–Charlie, Charlie–Dom, Charlie–Ethan.



TRY IT

Count the handshakes

How many people did each person shake hands with? Add those five numbers. There were only four handshakes, so why is the total different?

[Check the answers](#)

IMPORTANT RULE

The degree-sum rule

$$\text{sum of the degrees} = 2 \times \text{number of edges.}$$

Every edge meets two vertices, so every edge gets counted twice when we add all the vertex degrees.

Tip Learning tip

Before reading the next rule, see if you can predict it. You might want to use some scrap paper to draw a few small graphs and count how many odd-degree vertices each one has.

IMPORTANT RULE**Handshaking Lemma**

Every graph has an **even number of vertices of odd degree**.

PRACTICE**Practice puzzles**

- 1. Impossible party.** Nine people each claim that they shook hands with exactly three other people. Could everyone be telling the truth? Explain.
- 2. Draw your own.** Draw a graph with six vertices in which exactly four vertices have odd degree. Can you do it using exactly five edges?
- 3. Degree-sequence detective.** Could a graph have degree list 3, 2, 2, 1, 1? What tells you immediately?

Space for working

[Check the answers](#)

Tip Learning tip

For drawing problems, try arranging the vertices around a circle first. It often makes the connections easier to see and keeps edges from accidentally passing through other vertices.

CHALLENGE**Do degrees tell you everything?**

Can two different graphs have exactly the same list of vertex degrees? Draw an example or explain why not.

Space for working

[Check the answers](#)

Answers and discussion: Lesson 1

These are concise answers to the main questions. Open-ended drawing and design tasks usually have many possible solutions.

What is a graph?

Quick check

3 vertices and 2 edges.

Same graph?

Yes. Both drawings have the same four labelled vertices and exactly the same pattern of connections.

Handshake puzzle

Degrees are 2, 1, 3, 1, 1, totalling 8. There are 4 edges. Each handshake is counted once at each endpoint, so the degree total is twice the number of handshakes.

Impossible party

No. Nine people each reporting degree 3 would give total degree $9 \times 3 = 27$, but every degree sum must be even.

Degree list 3, 2, 2, 1, 1

Impossible: the degrees add to 9, which is odd.

Do degrees tell you everything?

No. For example, six vertices can all have degree 2 either as one six-cycle or as two separate triangles.